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ISOTHERMAL CYLINDRICAL SHOCK WAVES IN ROTATING ATMOSPHERE HAVING RADIATIVE HEAT FLUX

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Abstract

Chester-Chesnell-Whitham method has been applied to study the motion of spherical shock waves in self-gravitating heat conducting medium. This method is used to investigate the freely propagation of shock waves in uniform and non uniform atmosphere. Isothermal cylindrical shock waves in rotating atmosphere having radiative heat flux is investigated in this paper. The equilibrium flow conditions are assumed to be maintained and the radiation is considered to be of a diffusion type for an optically thick grey gas model. The electrical conductivity in the medium ahead of the shock is assumed to be negligible, which becomes infinitely large after passage of the shock. The density of the gas is assumed to be varying. The shock wave moves with variable velocity and the total energy of the wave is non constant. The present study can be important of description of shocks in supernova explosions, in the study of a flare produced shock in the solar wind, central part of star burst galaxies, nuclear explosion, repture of a pressurized vessel and explosion in the ionosphere etc. Neglecting the effect of overtaking disturbances, the expressions for shock velocity, shock strength, particle velocity, pressure, radiative heat flux are obtained.

KEY WORDS: Isothermal propagation, cylindrical shock, radiative flux, CCW method etc.

1. INTRODUCTION

Propagation of shock waves in a rotating atmosphere assumes special significance for research workers in the study of astrophysical phenomena. It has been observed that the outer atmosphere of the planets rotates due to the rotations of the planets. Macroscopic motion with supersonic speed occurs in interplanetary atmosphere with the rotation and shock waves are generated. Many scientists investigated shock waves in rotating atmosphere. Laumbach and Probstein (1970a)[4] have discussed the point explosion problem for radiating flow in an exponential atmosphere. Chaturani (1970)[5] studied the propagation of cylindrical shock waves through a gas having solid body rotation and obtained the solution by similarity method adopted by Sakurai (1956)[6]. Using similarity solution Verma and Singh (1980)[8] investigated the strong shock with radiation in a decreasing exponential atmosphere in presence of transverse magnetic field. Nath et al. (1999)[10] obtained the similarity solution for the flow behind spherical shock waves propagation in a non-uniform and rotating interplanetary atmosphere with increasing energy. Propagation of cylindrical shock waves in a non uniform rotating stellar atmosphere under the action of monochromatic radiation and gravitation. Propagation of cylindrical shock waves in a non-uniform rotating stellar atmosphere under the action of monochromatic radiation and gravitation is discussed by Yadav (2006)[15]. Nath (2010)[13] obtained the non similarity solution for the propagation of a strong shock waves in rotational axisymmetric dusty gas with exponentially varing density. Vishwakarma and Nath (2012)[14] have discussed the magneto gas dynamic shock waves in rotating gas with exponentially varying density. Arora and Sharma (2013)[9] discussed the similarity solution of cylindrical shock waves in magneto gas dynamic with thermal radiation. Vishwakarma and Patel (2015)[11] evaluated magneto gas dynamic cylindrical shock waves in a rotating non ideal gas with radiation heat flux. Very recently, Nath and Sahu (2016)[12] investigated similarity solution for the flow behind a cylindrical shock wave in a rotational axisymmetric gas with magnetic field and monochromatic radiation.

The aim of the present paper is to study the propagation of isothermal cylindrical shock waves in rotating atmosphere having radiative heat flux, assuming initial density distribution $\rho_0 \propto r^{\omega}$, where ω is density parameter.

2. CHESTER-CHISNELL WHITHAM METHOD:

A very well known method, Chester-Chisnell Whitham method is applied in this paper.

EQUATIONS OF MOTION AND BOUNDARY CONDITIONS:

The equations governing the flow of the gas enclosed by the shock front are

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial r}+\frac{\rho u}{r}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+\frac{u\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}-\frac{v^2}{r}=0 \tag{2}$$

$$\frac{\partial e}{\partial t}+\frac{u\partial e}{\partial r}-\frac{P}{\rho^2}\left[\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right]+\frac{1}{\rho r}\frac{\partial}{\partial r}(jr)=0 \tag{3}$$

where, $$e=\frac{p}{\rho(\gamma-1)}, p=\Gamma\rho T \tag{4}$$

where u, p, ρ, v, j and e are the radial component of velocity pressure, density, azimuthal component of velocity, monochromatic radiation flux and energy per unit mass at a radial distance r and time t respectively and γ is adiabatic index.

Strong shock conditions are given by:

$$u=\frac{2U}{(\gamma+1)}$$

$$\rho=\frac{\rho_0(\gamma+1)}{(\gamma-1)}$$

$$P=\frac{2\rho_0 U^2}{(\gamma+1)} \tag{5}$$

$$e=\frac{2U^2}{(\gamma+1)^2}$$

$$j=-\frac{\rho_0 U^3}{2}$$

SOLUTION

The characteristic equation is written as –

$$dp + \rho a du + \frac{\rho a}{u+a}\left[\frac{e(\gamma-1)^2}{ap}\frac{\partial}{\partial r}(jr) - v^2 + au'\right]\frac{dr}{r} = 0 \qquad (6)$$

When medium has density distribution $\rho_0 \propto r^{\omega}$ or $\rho_0 = b\, r^{\omega}$, where b is constant and ω is density parameter.

$$dp = \frac{2b}{(\gamma+1)}\left[U^2 \omega r^{\omega-1} dr + 2Ur^{\omega} dU\right]$$

$$\rho a du = \frac{\rho_0(\gamma+1)}{(\gamma-1)}\frac{a_0 2dU}{(\gamma+1)} = \frac{2\rho_0 a_0}{(\gamma-1)}dU$$

$$\frac{\rho a}{r(u+a)}\left[\frac{e(\gamma-1)^2}{ap}\frac{\partial}{\partial r}(jr)\right] = \frac{N\rho_0^{\alpha-\beta-4}(\gamma+1)^{\alpha-2\beta-7}2^{\beta+4}b^{2\beta+8}r^{\omega(\beta+4)}U^{2\beta+8}}{(\gamma-1)^{\alpha-\beta-4}[2U + a_0(\gamma+1)]}$$

$$\frac{\rho a v^2}{r(u+a)} = a_0\rho_0\Omega_0^2 r\frac{(\gamma+1)^2}{(\gamma-1)[2U + a_0(\gamma+1)]}$$

$$\frac{\rho a^2 u}{r(u+a)} = \frac{\rho_0(\gamma+1)}{r(\gamma-1)}\frac{a_0^2 2U}{(\gamma+1)}\frac{(\gamma+1)}{[2U + a_0(\gamma+1)]}$$

$$= \frac{2\rho_0 a_0^2 U(\gamma+1)}{r(\gamma-1)[2U + a_0(\gamma+1)]}$$

Putting all these values in equation (6) and solving we get –

$$\frac{dU}{dr} = -\left[\frac{N2^{\beta+4}b^{\alpha+\beta+3)}r^{\omega\alpha-\omega}U^{2\beta+8}(\gamma+1)^{\alpha-2\beta-7}}{(\gamma-1)^{\alpha-\beta-4}[2U + a_0(\gamma+1)]} + \frac{2U^2\omega}{r(\gamma+1)}\right.$$

$$\left. - \frac{a_0\Omega_0^2(\gamma+1)^2 r}{[2U + a_0(\gamma+1)](\gamma-1)} + \frac{2a_0^2 U(\gamma+1)\alpha'}{[2U + a_0(\gamma+1)]r(\gamma-1)}\right] \Big/ \left[\frac{4U}{(\gamma+1)} + \frac{2a_0}{(\gamma-1)}\right]$$

After computing above equation, the expression for shock velocity, shock strength, particle velocity, pressure, monochromatic raditive flux are determined.

3. RESULTS :

Results are obtained in the following forms –

$$U = -\int\left[\frac{N2^{\beta+4}b^{\alpha+\beta+3)}r^{\omega\alpha-\omega}U^{2\beta+8}(\gamma+1)^{\alpha-2\beta-7}}{(\gamma-1)^{\alpha-\beta-4}[2U+a_0(\gamma+1)]}+\frac{2U^2\omega}{r(\gamma+1)}\right.$$

$$\left.-\frac{a_0\Omega_0^2(\gamma+1)^2r}{[2U+a_0(\gamma+1)](\gamma-1)}+\frac{2a_0^2U(\gamma+1)\alpha'}{[2U+a_0(\gamma+1)]r(\gamma-1)}\right]dr\Big/\left[\frac{4U}{(\gamma+1)}+\frac{2a_0}{(\gamma-1)}\right]$$

$$\frac{U}{a_0} = -\int\left[\frac{N2^{\beta+4}b^{\alpha+\beta+3)}r^{\omega\alpha-\omega}U^{2\beta+8}(\gamma+1)^{\alpha-2\beta-7}}{(\gamma-1)^{\alpha-\beta-4}[2U+a_0(\gamma+1)]}+\frac{2U^2\omega}{r(\gamma+1)}\right.$$

$$\left.-\frac{a_0\Omega_0^2(\gamma+1)^2r}{[2U+a_0(\gamma+1)](\gamma-1)}+\frac{2a_0^2U(\gamma+1)\alpha'}{[2U+a_0(\gamma+1)]r(\gamma-1)}\right]dr\Big/\left[\frac{4U}{(\gamma+1)}+\frac{2a_0}{(\gamma-1)}\right]a_0$$

$$u = -\int\frac{2}{\gamma+1}\left[\frac{N2^{\beta+4}b^{\alpha+\beta+3)}r^{\omega\alpha-\omega}U^{2\beta+8}(\gamma+1)^{\alpha-2\beta-7}}{(\gamma-1)^{\alpha-\beta-4}[2U+a_0(\gamma+1)]}+\frac{2U^2\omega}{r(\gamma+1)}\right.$$

$$\left.-\frac{a_0\Omega_0^2(\gamma+1)^2r}{[2U+a_0(\gamma+1)](\gamma-1)}+\frac{2a_0^2U(\gamma+1)\alpha'}{[2U+a_0(\gamma+1)]r(\gamma-1)}\right]dr\Big/\left[\frac{4U}{(\gamma+1)}+\frac{2a_0}{(\gamma-1)}\right]$$

$$\frac{P}{P_0} = \frac{2\gamma}{a_0^2(\gamma+1)}\left[-\int\left[\frac{N2^{\beta+4}b^{\alpha+\beta+3)}r^{\omega\alpha-\omega}U^{2\beta+8}(\gamma+1)^{\alpha-2\beta-7}}{(\gamma-1)^{\alpha-\beta-4}[2U+a_0(\gamma+1)]}+\frac{2U^2\omega}{r(\gamma+1)}\right.\right.$$

$$\left.\left.-\frac{a_0\Omega_0^2(\gamma+1)^2r}{[2U+a_0(\gamma+1)](\gamma-1)}+\frac{2a_0^2U(\gamma+1)\alpha'}{[2U+a_0(\gamma+1)]r(\gamma-1)}\right]dr\Big/\left[\frac{4U}{(\gamma+1)}+\frac{2a_0}{(\gamma-1)}\right]\right]^2$$

$$j = \frac{br^{\omega}}{2}\left[\int\left[\frac{N2^{\beta+4}b^{\alpha+\beta+3)}r^{\omega\alpha-\omega}U^{2\beta+8}(\gamma+1)^{\alpha-2\beta-7}}{(\gamma-1)^{\alpha-\beta-4}[2U+a_0(\gamma+1)]}+\frac{2U^2\omega}{r(\gamma+1)}\right.\right.$$

$$\left.\left.-\frac{a_0\Omega_0^2(\gamma+1)^2r}{[2U+a_0(\gamma+1)](\gamma-1)}+\frac{2a_0^2U(\gamma+1)\alpha'}{[2U+a_0(\gamma+1)]r(\gamma-1)}\right]dr\Big/\left[\frac{4U}{(\gamma+1)}+\frac{2a_0}{(\gamma-1)}\right]\right]^3$$

4.DISCUSSION

The nature of flow variables are illustrated in table 1 to 5

Table 1 predicts that shock velocity, shock strength particle velocity, pressure increases while radiative heat flux decreases with propagation distance r.

Table 2 shows that shock velocity, shock strength, particle velocity, pressure decreases but radiative heat flux increases with adiabatic index γ.

Table 3 represents that shock velocity, shock strength, particle velocity, pressure decreases but radiative heat flux increases with temperature parameter β.

Table 4, it is noticed that shock velocity, shock strength, particle velocity, pressure decreases but radiative heat flux increases with density parameter ω.

Table 5 predicts that shock velocity, shock strength, particle velocity, pressure decreases but radiative heat flux increases with angular velocity Ω_0.

CONCLUSION

It is concluded that shock velocity, shock strength, particle velocity, pressure increases while radiative heat flux decreases as shock advances isothermally in heat radiating atmosphere but reverse trend is seen with other parameters. It is also concluded that results may be changed if the effect of overtaking disturbances is taken in to account.

TABLE 1

Variation of shock velocity, shock strength, particle velocity, pressure and radiative flux with propagation distance r for **cylindrical shock** (Initially taking U=1439.9626, a_0=330, r=11, γ=1.41, N=2.5, β=2, ω=1, α=1.5, b=1, Ω_0=1.5)

r	U	U/a_0	u	P/P_0	j
11	1439.9626	4.36352	1194.9897	22.4372	-41.5414
12	1454.8565	4.40865	1207.3497	22.9038	-42.8438
13	1469.7034	4.45364	1219.6708	23.3736	-44.1689
14	1485.5036	4.50152	1232.7830	23.8789	-45.6088
15	1503.6352	4.55647	1247.8300	24.4654	-47.2994
16	1523.6059	4.61698	1264.4032	25.1195	-49.2091
17	1544.8153	4.68125	1282.0043	25.8238	-51.2929
18	1567.3016	4.74939	1300.6652	26.5810	-53.5655
19	1592.6515	4.82621	1321.7024	27.4478	-56.2070
20	1619.8931	4.90876	1344.3096	28.3948	-59.1408

TABLE 2

Variation of shock velocity, shock strength, particle velocity, pressure and radiative flux with adiabatic index γ for **cylindrical shock** (Initially taking U=1547.4573, a_0=330, r=11, γ=1.43, N=2.5, β=2, ω=1, α=1.5, b=1, Ω_0=1.5)

γ	U	U/a_0	u	P/P_0	j
1.43	1547.4573	4.68926	1284.1969	25.9122	-51.5565
1.45	1538.0404	4.66072	1276.3821	25.5978	-50.6210
1.47	1529.1331	4.63373	1268.9901	25.3021	-49.7466
1.49	1520.8215	4.60855	1262.0925	25.0278	-48.9398
1.51	1512.6798	4.58387	1255.3359	24.7606	-48.1580
1.53	1502.0404	4.55163	1246.5065	24.4135	-47.1490
1.55	1491.3578	4.51926	1237.6413	24.0675	-46.1502
1.57	1479.1123	4.48215	1227.4790	23.6738	-45.0227
1.59	1465.1508	4.43985	1215.8927	23.2290	-43.7597

TABLE 3

Variation of shock velocity, shock strength, particle velocity, pressure and radiative flux with temperature parameter β for **cylindrical shock** (Initially taking U=1442.0835, a_0=330, r=11, γ=1.41, N=2.5, β=2.1, ω=1, α=1.5, b=1, Ω_0=1.5)

β	U	U/a_0	u	P/P_0	j
2.1	1442.0835	4.3699	1196.7497	22.5034	-41.7252
2.2	1436.6931	4.3536	1192.2764	22.3354	-41.2591
2.3	1429.1253	4.3306	1185.9960	22.1008	-40.6105
2.4	1421.4508	4.3074	1179.6272	21.8640	-39.9598
2.5	1414.8898	4.2875	1174.1824	21.6627	-39.4090
2.6	1406.3313	4.2616	1167.0799	21.4014	-38.6982
2.7	1398.2013	4.2369	1160.3330	21.1546	-38.0309
2.8	1392.2121	4.2188	1155.3627	20.9738	-37.5443
2.9	1385.9315	4.1979	1149.6526	20.7670	-36.9904

TABLE 4

Variation of shock velocity, shock strength, particle velocity, pressure and radiative flux with density parameter ω for **cylindrical shock** (Initially taking U=1646.0045, a_0=330, r=11, γ=1.41, N=2.5, β=2, ω=2, α=1.5, b=1, Ω_0=1.5)

ω	U	U/a_0	u	P/P_0	j
2	1646.0045	4.98789	1365.9788	29.3176	-62.0470
3	1634.8315	4.95403	1356.7066	28.9210	-60.7921
4	1622.7097	4.91730	1346.6470	28.4937	-59.4498
5	1609.1114	4.87609	1335.3621	28.0181	-57.9677
6	1596.3310	4.83736	1324.7560	27.5748	-56.5974
7	1582.7939	4.79634	1313.5219	27.1091	-55.1697
8	1569.8187	4.75702	1302.7541	26.6665	-53.8240
9	1557.7590	4.72048	1292.7460	26.2583	-52.5932

TABLE 5

Variation of shock velocity, shock strength, particle velocity, pressure and radiative flux with angular velocity Ω_0 for **cylindrical shock** (Initially taking U=1763.7225, a_0=330, r=11, γ=1.41, N=2.5, β=2, ω=1, α=1.51, b=1, Ω_0=1.5)

Ω_0	U	U/a_0	u	P/P_0	j
1.51	1763.7225	5.34461	1463.6701	33.6610	-76.3341
1.52	1682.0142	5.09701	1395.8624	30.6144	-66.2090
1.53	1601.3229	4.85249	1328.8986	27.7475	-57.1300
1.54	1517.9986	4.59999	1259.7498	24.9350	-48.6666
1.55	1433.6668	4.34444	1189.7649	22.2414	-40.9989
1.56	1343.3430	4.07073	1114.8074	19.5272	-33.7278
1.57	1248.1505	3.78227	1035.8095	16.8578	-27.0538
1.58	1143.7445	3.46589	949.16555	14.1555	-20.8168
1.59	1033.8779	3.13296	857.98995	11.5666	-15.3757

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